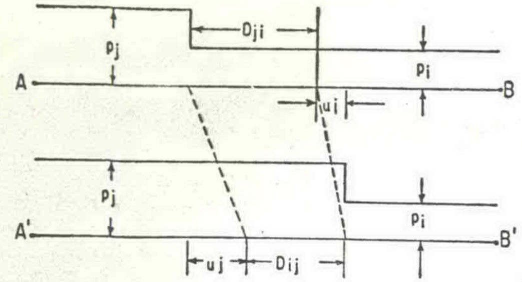

FIG. 2. Sketch showing progress of the pressure profile of a shock in unit time. Propagation velocity and particle velocities are indicated.

The change of momentum occurring in this mass is

$$p_j - p_i = \frac{D_{ji}}{v_i}(u_j - u_i). \qquad (3)$$

Elimination of $(u_j - u_i)$ between Eqs. (2) and (3) leads at once to Eq. (1). If one desires the propagation velocity D_{ij} with respect to material behind the shock, it can be expressed as

$$D_{ij} = v_j\left(\frac{p_j - p_i}{v_i - v_j}\right)^{\frac{1}{2}}. \qquad (4)$$

In all the foregoing, stability of the system depicted in Fig. 1 has been assumed. Though a detailed discussion of the stability of compressive shock fronts is beyond the scope of the present paper, it should be clear that a necessary condition for the stable propagation of more than one discontinuous pressure jump is that the velocity of propagation of the first shock with respect to the material behind it exceed the velocity of propagation of the second shock with respect to the material ahead of it. A hypothetical pressure-volume relationship sufficient to permit such a situation appears as a solid line in Fig. 3. The dotted lines merely serve to connect states of the metal on different sides of various possible shock fronts. Comparison of Eq. (1) with Eq. (4) reveals that the slopes of two dotted lines having a common point provide an adequate criterion for judging whether or not the corresponding shocks will separate.

If the ultimate pressure and volume correspond to point E, then only one stable discontinuity would be possible, the propagation velocity being 4.9. If point D represents the ultimate conditions, two shocks are possible, whereas point C would be reached by three shocks.

In a qualitative way, Fig. 3 represents the behavior believed to be characteristic of iron. It should be remembered that compression of a metal by a plane wave in an infinite medium occurs without lateral displacement of the particles. At small compressions the relation between longitudinal stress Δp and volume v is adequately described by Hooke's law in the form

$$\frac{\Delta v}{v} = \frac{\Delta p}{\kappa + \frac{4}{3}\mu}, \qquad (5)$$

where κ is the bulk modulus and μ the shear modulus, there being a very considerable shearing stress developed, the magnitude of which is

$$\Delta\tau = \frac{2\mu}{3\kappa + 4\mu}\Delta p. \qquad (6)$$

There is in general a maximum shearing stress that the metal will support, though this maximum may depend on temperature, pressure, and rate of loading. As the longitudinal stress is increased, the value of $\Delta\tau$ as specified by Eq. (6) may be expected to reach this maximum value, and thereafter to increase very slightly or not at all (or perhaps even to vanish altogether). This is observed as a "yielding" of the metal, and if $\Delta\tau$ becomes a small fraction of Δp, the dilatation of the metal approaches the value it would have under a high hydrostatic pressure numerically equal to the longitudinal stress Δp, *viz.*

$$\frac{\Delta v}{v_0} = \frac{\Delta p}{\kappa} + f(\Delta p), \qquad (7)$$

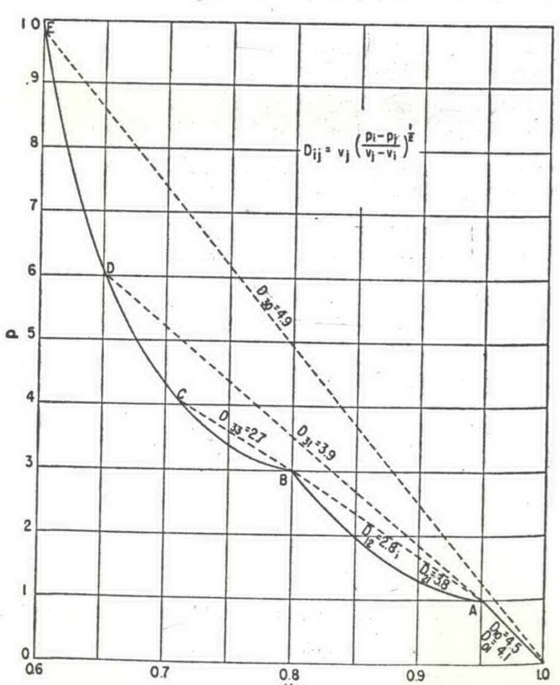

FIG. 3. Pressure-volume relationship showing qualitatively the features believed to be characteristic of iron. The effects have been exaggerated for the purpose of clarifying the determinations of various shock velocities. Units of P, V, and D are arbitrary, but consistent.

where $f(\Delta p)$ is an empirical function involving powers of Δp higher than the first. In Fig. 3, this yielding is shown as occurring abruptly at point A. Actually there would probably be no sharp break, though the details are obscure, not only experimentally but also theoretically.

A second yield point or cusp is shown at B (Fig. 3). It would hardly be reasonable to assume this second yielding to be due to failure of the metal to support shearing stresses, for these have already been assumed to be negligibly small in comparison to Δp. The most reasonable supposition as to the origin of such a point is that a polymorphic transition occurs from α iron to γ iron. At atmospheric pressure this transition occurs at 910°C, with absorption of heat and contraction of volume. Bridgman's[6] investigations of such phenomena as induced by pressure show in general an abrupt volume change without increase of pressure. However, his experiments are made at constant temperature. If a transformation were to occur at constant entropy, not all the material would transform at once, and the pressure-volume relation for the system of mixed phases would be qualitatively that shown by the solid curve beyond B in Fig. 3. In this case, a compressive shock system as depicted in Fig. 1 becomes possible.

In order to determine changes in pressure and specific volume from Eqs. (2) and (3), one must determine both propagation velocity D and particle velocity u. The latter is inferred from the free surface velocity $(\sigma + u)$, and may generally be taken to be one-half the latter. The necessary calculations, including the relation between free-surface and particle velocity, have been discussed by Goranson.[4] In our notation [see Eq. (16) *infra*], his Eq. (10) may be expanded in the approximate form

$$\frac{\sigma - u}{u} \doteq \frac{1}{24}\left(1 + \frac{\beta_s}{\alpha_s}\right)\left(\frac{2}{\varphi} - \frac{\beta_s}{\sigma_s} - 1\right)\mu_D{}^2, \qquad (8)$$

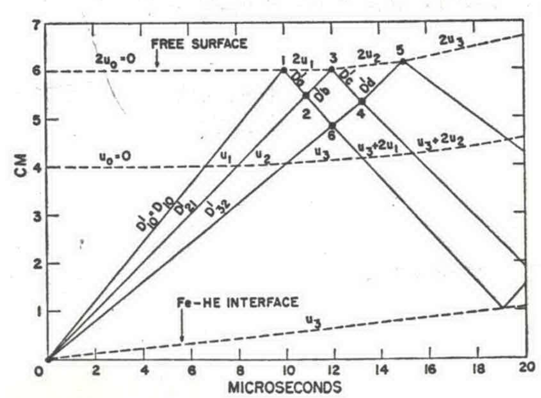

FIG. 4. Propagation of shocks into material originally at rest, showing also the reflection of the shocks from a free surface. Solid lines represent progress of shocks, dotted lines of individual particles. Lines are labeled with the velocities to which their slopes correspond. Drawing is approximately to scale for iron 6 cm thick.

[6] P. W. Bridgman, *Physics of High Pressure* (G. Bell and Sons, Ltd., London, 1949), p. 421.

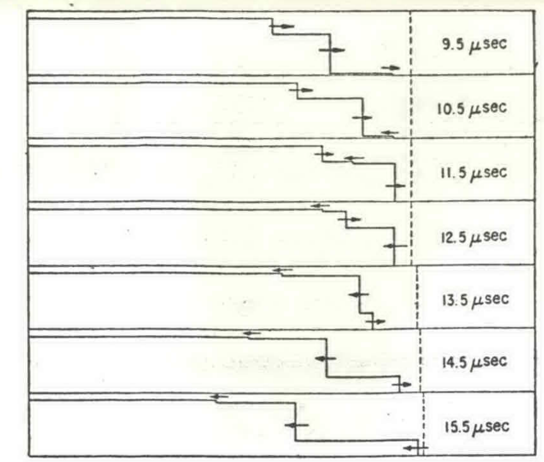

FIG. 5. Pressure profiles of compressive wave with three discrete shocks being reflected from a free surface. Rarefactions also shown as discontinuities, though actually these would become ogives. The times and positions in this figure correspond to those in Fig. 4. Arrows denote direction in which the various wave fronts are progressing.

where $\varphi = (C_v/v_0)(\partial T/\partial p)_v$ is an intrinsic dimensionless property of the material, and μ_D is the compression reached by the shock. Equation (8) implies the possibility of either sign for $(\sigma - u)$. Since the properties of the material enter this equation in such a way as to tend to cancel one another, it seems likely that in many cases a higher order of approximation will be necessary in order to obtain an accurate result. But in most cases Eq. (8) should at least suffice to determine whether or not the approximation $\sigma = u$ is justifiable. In the case of iron, for $p_D = 0.13$ megabar, one finds $(\sigma - u)/u = 0.0001$, which for our purposes is negligibly small.

If, however, there is a break in the pressure-volume relationship for iron, then computation of the correction after the material has been compressed beyond the break requires data not currently available, for in this case the adiabatic path along which the material expands is largely a matter of conjecture. It will be evident from Goranson's[4] derivation of his Eq. (10) that the correction cannot be determined without this information. The assumption that $u = \sigma$ should thus be regarded as a useful approximation only.

Figure 4 shows, as functions of time, positions of shock fronts as solid lines, and positions of particles as broken lines. In accordance with the discussion above, all free surface velocities are shown as twice the mass velocities existing behind the shock before reflection occurred. The various lines are labeled with the velocities to which their slopes correspond. The fact that all shock velocities are shown relative to the laboratory system is indicated by accents, so that

$$D_{ij} + u_j = D_{ij}' = D_{ji}' = D_{ji} + u_i. \qquad (9)$$

Reflection and refraction of rarefactions by shocks have been ignored. The velocities with which the various rarefactions recede have been labeled D_a', D_b', D_c', and